

OPERATOR RELATIVES AND HYPERINVARIANCE

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ABSTRACT. We show that every non-zero weakly compact operator on a complex Banach space admits a hyperinvariant subspace. Additionally, if each non-scalar algebraic operator admits a finite-dimensional eigenspace, then every non-scalar operator has a hyperinvariant subspace when its commutant (or, bicommutant) has a non-scalar intersection with the bicommutant (respectively, commutant) of a non-zero weakly compact operator. These results extend, *mutatis mutandis*, to real Banach spaces. Our proof refines the approach of Lomonosov-Sirotkin for compact operators by exploiting the weak compactness of the operator and the weak topology of the space, and we employ a dentability theorem of Troyanski, the Krein-Šmulian theorem, and the Schauder-Tikhonov theorem.

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